Chapter – 12

Herons Formula

Area of Triangle by Herons Formula

Perimeter: Perimeter of a shape can be defined as the path or the boundary that surrounds the shape. It can also be defined as the length of the outline of a shape.

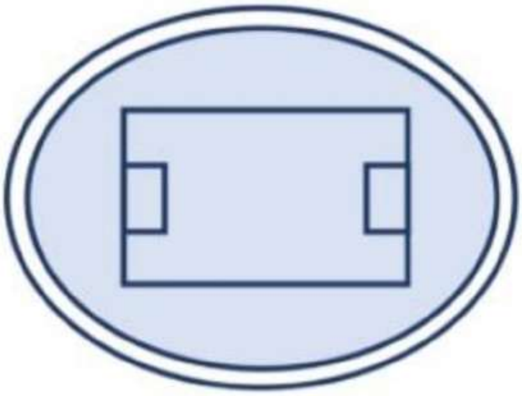
The word perimeter has been derived from the Greek word 'peri' meaning around, and 'metron' which means measure.

How to find Perimeter

During Christmas, we find people decorating their homes. For example, people put up decorating lights around their homes like around the fences of their homes. To find what length of lighting is required, we have to find the perimeter of the fencing. We often find the perimeter when putting up Christmas lights around the house or fencing the field.

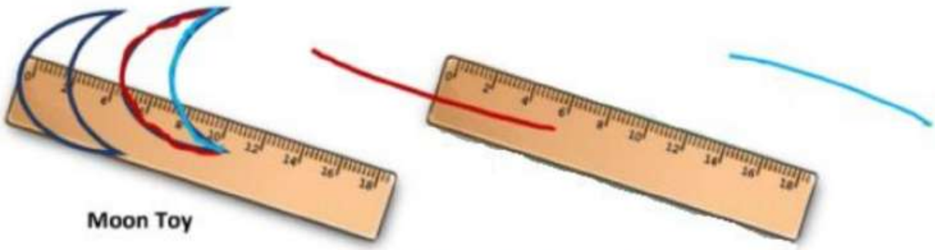
Christmas lights fencing the field

If we have to find the length of the track around the oval-shaped ground, then we have to find the perimeter of the ground.



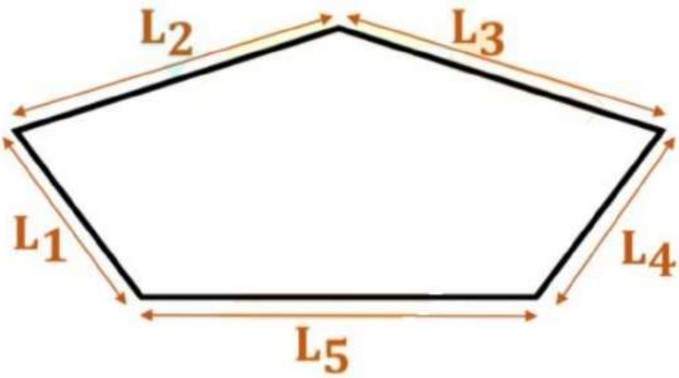
Soccer Field

For measuring the perimeter of small irregular shape, we can use a string or thread and place it exactly along the boundary of the shape, once. Then keeping the thread along the ruler, we find the length of threads. The total length of the string used along the boundary is the perimeter of the shape.



Perimeter of moon toy = 12 cm + 10 cm= 22 cm.

We use a ruler to measure the length of the sides of a polygon. The perimeter is determined by adding the lengths of the sides/edges of the shape.



Perimeter = $L_1 + L_2 + L_3 + L_4 + L_5$

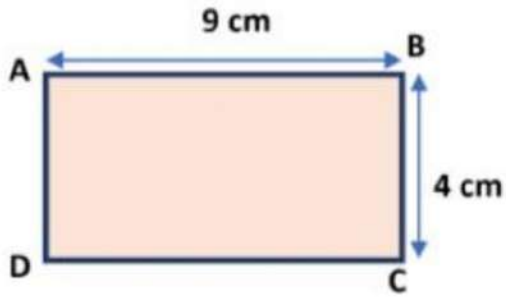
Area: Area of an object can be defined as the space occupied by the two-dimensional object. The area of a figure is the number of unit squares that cover the surface of a closed figure.

The area is measured in square units such as square centimeters, square feet, square inches, etc.

Area of rectangle ABCD = Length × Breath.

$= 9 \text{ cm} \times 4 \text{ cm}.$
$= 36 \text{ cm}^2.$

Perimeter of rectangle ABCD = AB + BC + CD + DA.
$= 9 \text{ cm} + 4 \text{ cm} + 9 \text{ cm} + 4 \text{ cm}.$
$= 26 \text{ cm}$



For finding the area of a right-angled triangle, we can directly use the formula

(Area of triangle = $\frac{1}{2}$ (base x height)

We use the two sides containing the right angle as base and height.

Example 1: Find the area of right triangle ABC whose base and height are 5 cm and 8 cm respectively.

Solution: Since, ΔABC is a right-angled triangle, in which height AD = 8 cm and base CB = 5 cm.

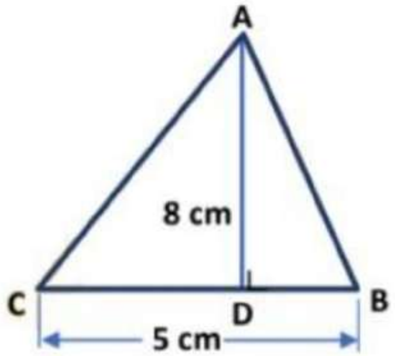
Using the above formula, we can write

Area of right-angled triangle ABC = $\frac{1}{2}$ (base × height).

$= \frac{1}{2}(5\text{ cm}\times 8\text{ cm})$.

$= (5\text{ cm}\times 4\text{ cm})$

$= 20\text{ cm}^2$.



Base is the side on which the height (perpendicular) is drawn. Height is the perpendicular drawn from opposite vertex to its base.

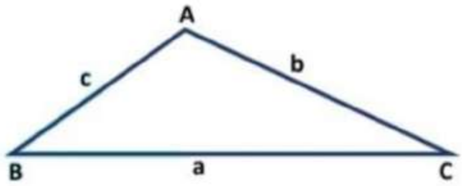
When all 3 sides of a triangle are known, to calculate its area, we need to find its height. But finding its height may be tedious. In this case, we use Heron's formula to find the area of the triangle in geometry.

This formula makes the calculation of finding the area of a triangle simple by eliminating the use of angles and the need for the height of the triangle.

The formula given by Heron about the area of a triangle is known as Heron's formula. It is stated as:

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

Conventionally: Length of opposite side of a vertex denoted by its small letter. For Example: Length of opposite side of vertex A is denoted by



Where a, b and c are the sides of the triangle, and S = semi – perimeter, i.e., half the perimeter of the triangle = $\frac{a+b+c}{2}$.

Example2: Find the area of an equilateral triangle ABC with the side "a".

Solution: Since ABC is an equilateral triangle, So, all sides are equal.

That is, AB = BC = CA = a.

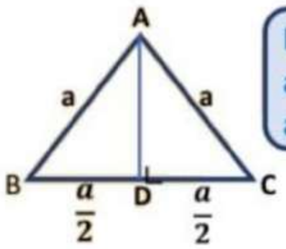
By Pythagoras theorem,

We can write, $AB^2 = BD^2 + AD^2$.

$AD^2 = AB^2 - BD^2$.

$= a^2 - (\frac{a}{2})^2 = a^2 - (\frac{a^2}{4}) = \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$.

AD = $\frac{\sqrt{3}a}{2}$



In equilateral triangle, attitude from the vertex also bisects the base.

Therefore, in equilateral triangle ABC, base BC = a and height AD = $\sqrt{3} \times \frac{a}{2}$.

So, the area of an equilateral triangle = $\frac{1}{2}(a \times \frac{\sqrt{3}a}{2}) = \frac{\sqrt{3}a^2}{4}$.

OR

This same question is also solved by Heron's Formula.

And, s = $\frac{a+a+a}{2} = \frac{3a}{2}$.

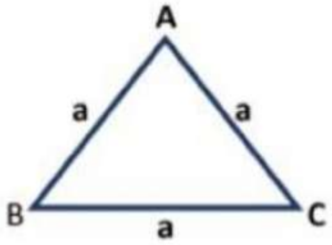
By Heron's Formula, we can write

Area of an equilateral triangle =

$=\sqrt{\frac{3a}{2}(\frac{3a}{2}-a)(\frac{3a}{2}-a)(\frac{3a}{2}-a)}$

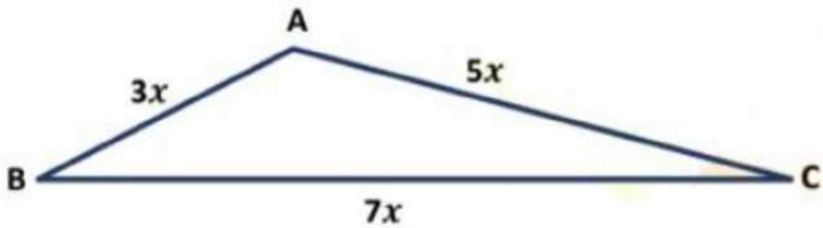
$=\sqrt{\frac{3a}{2}(\frac{a}{2})(\frac{a}{2})(\frac{a}{2})}$

$= \frac{\sqrt{3}a^2}{4}$ (Where a is the length of the side)



Area of an equilateral triangle = $\sqrt{3}\frac{a^2}{4}$

Example3: The sides of a triangular plot are in the ratio of 3:5:7 and its perimeter is 450 m. Find its area.



Solution: Since sides of a triangular plot are in the ratio 3:5:7. For exact values, they contain a common factor x, which we have to find.

Let AB = 3x, AC = 5x and BC = 7x.

By definition of perimeter,

Perimeter of triangular plot = AB + BC + AC.

$$450 = 3x + 5x + 7x.$$

$$\Rightarrow \quad 450 = 15x.$$

$\Rightarrow x = \frac{450}{15}$.

$\Rightarrow x = 30.$

Therefore, AB = 3x = 3 × 30 m = 90 m.

AC = 5x = 5 × 30 m = 150 m.

BC = 7x = 7 × 30 m = 210 m.

Now, semi-perimeter of triangular park = $\frac{450m}{2}$.

$$S = 225 \text{ m}.$$

So, the area of triangular park = $\sqrt{s(s-a)(s-b)(s-c)}$

[∵ Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$]

$= \sqrt{225 \text{ m } (225 \text{ m} - 210 \text{ m})(225 \text{ m} - 150 \text{ m})(225 \text{ m} - 90 \text{ m})}$

$= \sqrt{225 \text{ m} \times 15 \text{ m} \times 75 \text{ m} \times 135 \text{ m}}$.

$= \sqrt{5 \times 5 \times 3 \times 3 \times 5 \times 3 \times 5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3}\ m^2$

$= \sqrt{5 \times 5 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3}\ m^2$

$= 5 \times 3 \times 5 \times 5 \times 3 \times 3\sqrt{5} m^2$

$= 5 \times 3 \times 5 \times 5 \times 3 \times 3\sqrt{5} m^2$

$= 3{,}375\sqrt{5}m^2$

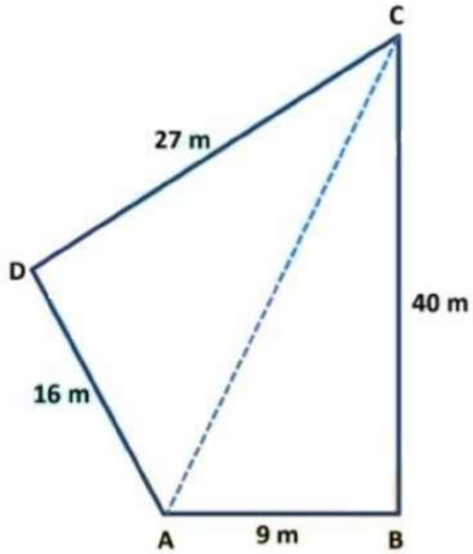
$= 3375 \times 2.2360\ m^2\ [\because \sqrt{5} = 2.2360]$

$= 7546.5\ m^2$

Hence, area of triangular park = $7546.5\ m^2$

Herons Formula in Finding area of Quadrilaterals

Example: Find the area of a quadrilateral ABCD whose sides are 9m, 40 m, 27 m, and 16 m respectively and the angle between the first two sides is a right angle.



Solution: Let ABCD be the given quadrilateral such that $\angle ABC = 90°$ and AB = 9m, BC = 40m, CD = 28m, AD = 15m.

In Δ ABC, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AC^2 = 9^2m^2 + 40^2m^2$ (By Pythagoras theorem)

$\Rightarrow AC^2 = 81\ m^2 + 1600\ m^2$

$\Rightarrow AC^2 = 1681\ m^2$

$\Rightarrow AC = \sqrt{1681}$ m, AC = 41 m.

Now, Area of Δ ABC = $\frac{1}{2}$ (base x height)

$\Rightarrow$Area of Δ ABC = $\frac{1}{2}$ (AB x BC)

$\Rightarrow$Area of Δ ABC = $\frac{1}{2}$ (9m x 40m).

$\Rightarrow$Area of Δ ABC = $\frac{1}{2}$ (360)m^2.

$\Rightarrow$Area of Δ ABC = 180m^2.

In Δ ACD, we have

AC =41m, CD = 27 m and DA = 16 m

Let a = AC = 41m, b = CD = 27m and c = DA = 16m.

Then, s = $\frac{1}{2}$ (a + b + c).

$\Rightarrow$s = $\frac{1}{2}$ (41m + 27m + 16m).

$\Rightarrow$s = $\frac{1}{2}$ (84m) = 42m.

Therefore, the area of Δ ACD = $\sqrt{s(s-a)(s-b)(s-c)}$.

ar(Δ ACD) =$\sqrt{42m(42m - 41m)(42m - 27m)(42m - 16m)}$.

$= \sqrt{42m(1m)(15m)(26m)}$.

$= \sqrt{2 \times 3 \times 7 \times 3 \times 5 \times 2 \times 13m^2}$.

$= \sqrt{2 \times 2 \times 3 \times 3 \times 7 \times 5 \times 13m^2}$.

$= 6\sqrt{455}\ m^2$

$= 127.98 \text{ m}^2$.

$\therefore$ Area of quadrilateral ABCD = (Area of Δ ABC) + (Area of Δ ACD).

$= 180\text{m}^2 + 127.98 \text{ m}^2$.

$= 307.98 \text{ m}^2$.

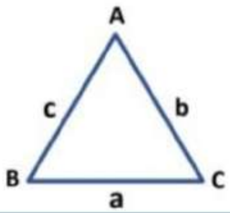
Summary of Herons Formula

Heron's Formula

Area of a triangle with its sides as a, b and c is calculated by using Heron's Formula stated as

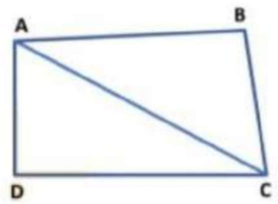
Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where, S = semi - perimeter = $\frac{a+b+c}{2}$.



Applications of Heron's Formula

Area of a quadrilateral whose sides and one diagonal are given, can be calculated by dividing the quadrilateral into two triangles and then using the Heron's Formula to find the area of the two triangles thus formed.



Then we add the area of two triangles to get the area of the quadrilateral.